Magnetic Fields Boosted by Gluon Vortices in Color Superconductivity

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We investigate the effects of an external magnetic field in the gluon dynamics of a color superconductor with three massless quark flavors. In the framework of gluon mean-field theory at asymptotic densities, we show that the long-range component $\widetilde{H}$ of the external magnetic field that penetrates the CFL phase produces an instability when its strength becomes larger than the Meissner mass of the charged gluons. As a consequence, the magnetic field causes the formation of a vortex state characterized by the condensation of charged gluons and the creation of magnetic flux tubes. Inside the flux tubes the magnetic field is stronger than the applied one. This antiscreening effect is connected to the anomalous magnetic moment of the gluon field. We suggest how this same mechanism could serve to remove the chromomagnetic instabilities existing in gapless color superconductivity.

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It is quite plausible [1] that color superconductivity (CS) occurs in the inner regions of neutron stars, hence affecting the structure and properties of these compact objects. At the same time, it is known that strong magnetic fields exist in the surface of compact stars, reaching values of orders $10^{14} - 10^{16}$ G in the case of magnetars [2]. Comparing magnetic and gravitational energies it has been shown [3] that the physical upper limit for the total neutron star magnetic field is $B \sim 10^{18}$ G. If quark stars indeed exist and are self-bound rather than gravitational-bound this upper limit can be even higher. Therefore, the investigation of CS in the presence of a magnetic field is of main interest for possible astrophysical applications.

It is well established by now that at densities high enough to neglect the strange quark mass, the ground state of three-flavor quark matter corresponds to the color-flavor locked (CFL) phase [4]. In this phase, quarks form spin-zero Cooper pairs in the color-antitriplet, flavor-antitriplet representation. An important feature of spin-zero color superconductivity is that although the color condensate has non-zero electric charge, the linear combination $\widetilde{A}_\mu = \cos\theta\, A_\mu + \sin\theta\, G^8_\mu$ of the photon A_μ and the gluon G^8_μ remains massless [4, 5], so it behaves as an in-medium electromagnetic field, while the orthogonal combination $\widetilde{G}^8_\mu = -\sin\theta A_\mu + \cos\theta\, G^8_\mu$ is massive. In the CFL phase the mixing angle θ is small thus the penetrating "electromagnetic" field is mostly formed by the original photon with only a small gluon admixture.

Even though gluons are electrically neutral, in the CFL phase they can interact with the modified (also called rotated) photon through their $\widetilde{Q}$ charges:

G^1_μ	G^2_μ	G^3_μ	G^+_μ	G^-_μ	I^+_μ	I^-_μ	$\widetilde{G}^8_\mu$
0	0	0	1	-1	1	-1	0

, (1)

given in units of $\widetilde{e} = e\cos\theta$. The $\widetilde{Q}$-charged fields in (1) correspond to the combinations $G^\pm_\mu \equiv \frac{1}{\sqrt{2}}[G^4_\mu \pm iG^5_\mu]$ and $I^\pm_\mu \equiv \frac{1}{\sqrt{2}}[G^6_\mu \pm iG^7_\mu]$. If the color superconductor is penetrated by a sufficiently strong rotated magnetic field, the structure and magnitude of the CFL gap are modified giving rise to a new phase called Magnetic CFL (MCFL) [6]. The MCFL [6] was obtained in the context of an effective NJL model in which the interactions between the $\widetilde{Q}$-charged gluons and the magnetic field were ignored. In this letter, we are interested in extending those previous investigations by considering the impact of an external magnetic field on the gluon dynamics. We will show that at high enough magnetic field there is an inhomogeneous condensate of the $\widetilde{Q}$-charged gluons that antiscreens the magnetic field due to the anomalous magnetic moment of these spin-1 particles. As a consequence, this condensate does not give a mass to the $\widetilde{Q}$ photon, but instead it amplifies the applied $\widetilde{Q}$ magnetic field (for the analogous behavior of W bosons in a high magnetic field, see [7]).

We will work in the weak coupling region of QCD, assuming that the external magnetic field is much smaller than the square of the chemical potential, so that the effects of the field on the gap structure and magnitude can be neglected. In this case, it is safe to assume that the superconductor is in the CFL phase.

We start from the gauge field sector of the mean-field effective action in the CFL phase at asymptotic densities. This effective action, which includes the one-loop quark contribution obtained after integrating out the Nambu-Gorkov fields, can be written as

$$\begin{aligned}\Gamma_{eff} &= -\frac{1}{4}\int d^4x[(G^a_{\mu\nu})^2 + (f_{\mu\nu})^2] + \int d^4x L_g(x) \\ &- \frac{1}{2}\int d^4x d^4y G^a_\mu(x)\Pi^{ab}_{\mu\nu}(x,y)G^b_\nu(y) \\ &- \frac{1}{2}\int d^4x d^4y A_\mu(x)\Pi_{\mu\nu}(x,y)A_\nu(y) \qquad (2)\end{aligned}$$

where L_g is the gauge fixing Lagrangian density, $G^a_{\mu\nu}$ and $f_{\mu\nu}$ are the gluon and electromagnetic field strength tensors respectively, and $\Pi^{ab}_{\mu\nu}$ and $\Pi_{\mu\nu}$ are their corresponding polarization operators in the CFL phase [8]. No mixing between the gauge fields and the Nambu-Goldstone mesons is included because this mixing can be eliminated using the 't Hooft gauge [9].

Since we are interested in the CFL-system response to a constant external magnetic field, we can concentrate our attention into the $\widetilde{Q}$-charged gluon part of (2) in the presence of an external rotated magnetic field $\widetilde{H}$. Without loss of generality, the analysis can be done for one of the two sets of charged fields, say $G_\mu^\pm$. For the static response, one only needs the leading contribution of the polarization operators [8] of the $\widetilde{Q}$-charged gluons in the infrared limit ($p_0 = 0, |\overrightarrow{p}| \to 0$): $\Pi_{\mu\nu}^{ab}(x, y) = [m_D^2 \delta_{\mu 0}\delta_{\nu 0} + m_M^2 \delta_{\mu i}\delta_{\nu i}]\delta(x-y)\delta^{ab}$, with a,b=4,...,7. The Debye (m_D) and Meissner (m_M) masses are given by $m_D^2 = \sigma m_g^2$, $m_M^2 = \sigma m_g^2/3$, with $\sigma = (21 - 8\ln 2)/18$, and $m_g^2 = g^2(\mu^2/2\pi^2)$. We neglect the possible corrections produced by the applied field in the infrared masses since for fields weak compared to the quark chemical potential, it is a second order effect. The effective action for the fields $G_\mu^\pm$ in the background of the $\widetilde{H}$ external field becomes

$$\Gamma_{eff}^c = \int d^4x\{-\tfrac{1}{4}(\widetilde{f}_{\mu\nu})^2 - \tfrac{1}{2}|\widetilde{\Pi}_\mu G_\nu^- - \widetilde{\Pi}_\nu G_\mu^-|^2 \\ -[(m_D^2\delta_{\mu 0}\delta_{\nu 0} + m_M^2\delta_{\mu i}\delta_{\nu i}) + i\widetilde{e}\widetilde{f}_{\mu\nu}]G_\mu^+ G_\nu^- \\ +\tfrac{g^2}{2}[(G_\mu^+)^2(G_\nu^-)^2 - (G_\mu^+ G_\mu^-)^2] + \tfrac{1}{\lambda}G_\mu^+\widetilde{\Pi}_\mu\widetilde{\Pi}_\nu G_\nu^-\}, \quad (3)$$

where the last term in (3) comes from the L_g term in (2), taken in the 't Hooft gauge with arbitrary gauge fixing parameter λ, $\widetilde{\Pi}_\mu = \partial_\mu - i\widetilde{e}\widetilde{A}_\mu$, and $\widetilde{f}_{\mu\nu} = \widetilde{f}_{12}^{ext} = \widetilde{H}$, that is, we take the external rotated magnetic field along the third spatial direction. The effective action (3) is nothing but the characteristic effective action of a spin-1 charged boson in a magnetic field [10]. As known, due to the anomalous magnetic moment term ($i\widetilde{e}\widetilde{f}_{\mu\nu}G_\mu^+ G_\nu^-$), when the field surpasses a critical value, one of the modes of the charged gauge field becomes tachyonic (this is the well known "zero-mode problem" found in the presence of a magnetic field for Yang-Mills fields [11], for the $W_\mu^\pm$ bosons in the electroweak theory [7, 12], and even for higher-spin fields in the context of string theory [13]). The tachyonic mode can be easily found from (3) diagonalizing the mass matrix of the field components $(G_1^\pm, G_2^\pm)$

$$\begin{pmatrix} m_M^2 & i\widetilde{e}\widetilde{H} \\ -i\widetilde{e}\widetilde{H} & m_M^2 \end{pmatrix} \to \begin{pmatrix} m_M^2 + \widetilde{e}\widetilde{H} & 0 \\ 0 & m_M^2 - \widetilde{e}\widetilde{H} \end{pmatrix} \quad (4)$$

Clearly, above the critical field $\widetilde{e}\widetilde{H}_C = m_M^2$ the lowest mass mode in (4) becomes tachyonic, with corresponding eigenvector of amplitude G in the $(1, i)$ direction for G^- (G^* in the $(1, -i)$ direction for G^+). We emphasize that the $\widetilde{H}$ field producing the instability is the in-medium long-range field that does not suffer the Meissner effect in the CFL phase.

Similarly to other spin-1 theories with magnetic instabilities [7]-[11], the solution of the zero-mode problem leads to the restructuring of the ground state through the formation of a gauge field condensate G, as well as an induced magnetic field $\widetilde{\mathbf{B}} = \nabla \times \widetilde{\mathbf{A}}$ due to the backreaction of the G condensate on the rotated electromagnetic field.

The condensate solutions can be found by minimizing the Gibbs free energy density $\mathcal{G}_c = \mathcal{F} - \widetilde{H}\widetilde{B}$, ($\mathcal{F}$ is the free energy density), with respect to G and $\widetilde{B}$. Since an applied field $\widetilde{H} > \widetilde{H}_C$ in the third direction develops an instability in the (x, y)-plane for the eigenmode $G(1, i)$, it is reasonable to make the ansatz $G_1^- = -iG_2^- = G(x, y)$, $G_3^- = G_0^- = 0$ and $G_i^+ = (G_i^-)^*$. Considering this ansatz and fixing the gauge parameter to $\lambda = 1$ in the action (3) we obtain for the Gibbs free energy density in the G-condensate phase

$$\mathcal{G}_c = \mathcal{F}_{n0} - 2G^\dagger\widetilde{\Pi}^2 G - 2(2\widetilde{e}\widetilde{B} - m_M^2)|G|^2 + 2g^2|G|^4 \\ +\frac{1}{2}\widetilde{B}^2 - \widetilde{H}\widetilde{B} \quad (5)$$

where $\mathcal{F}_{n0}$ is the system free energy density in the normal-CFL phase ($G = 0$) at zero applied field.

Using (5) the minimum equations for the condensate G and induced field $\widetilde{B}$ respectively are

$$-\widetilde{\Pi}^2 G - (2\widetilde{e}\widetilde{B} - m_M^2)G + 2g^2|G|^2 G = 0, \quad (6)$$

$$2\widetilde{e}|G|^2 - \widetilde{B} + \widetilde{H} = 0 \quad (7)$$

Identifying G with the complex order parameter, Eqs. (6)-(7) become analogous to the Ginzburg-Landau equations for a conventional superconductor except for the $\widetilde{B}$ contribution in the second term in (6) and the sign of the first term in (7). The origin of both terms can be traced back to the anomalous magnetic moment term in the action of the charged gluons. Notice that because of the different sign in the first term of (7), contrary to what occurs in conventional superconductivity, the resultant field $\widetilde{B}$ is stronger than the applied field $\widetilde{H}$. Thus, when a gluon condensate develops, the magnetic field will be antiscreened and the CS will behave as a paramagnet. The antiscreening of a magnetic field has been also found in the context of the electroweak theory for magnetic fields $eH \geq M_W^2$ [7]. Just as in the electroweak case, the antiscreening of the CS is a direct consequence of the asymptotic freedom of the underlying theory [7, 14].

The main goal of this paper will be to investigate the qualitative features of the new phase with the gluon condensate at $\widetilde{H} \simeq \widetilde{H}_C$. A starting point in this direction will be to investigate the sign of the condensation energy

$$\alpha_{nc} = \int_{-\infty}^{\infty}[\mathcal{G}_c - \mathcal{G}_n]dx = \int_{-\infty}^{\infty}\{-2G^\dagger\widetilde{\Pi}^2 G + \frac{1}{2}(\widetilde{B} - \widetilde{H}_C)^2 \\ -2(2\widetilde{e}\widetilde{B} - m_M^2)|G|^2 + 2g^2|G|^4\}dx \quad (8)$$

with $\mathcal{G}_n = \mathcal{F}_{n0} - \widetilde{H}_C^2/2$ being the Gibbs free energy density of the normal-CFL phase in the external rotated magnetic field $\widetilde{H}_C$. Considering that just below $\widetilde{H}_C$

the system is in the normal-CFL phase, we should look for the possibility of nucleation at $\widetilde{H}_C$ of an inhomogeneous condensate satisfying asymptotic boundary conditions that correspond to the normal-CFL phase, i.e. $G(x \to \pm\infty) = 0$. Using the Ginzburg-Landau equations (6)-(7) we can rewrite (8) as

$$\alpha_{nc} = \int_{-\infty}^{\infty} (1 - \frac{g^2}{\widetilde{e}^2}) \frac{(\widetilde{B} - \widetilde{H}_C)^2}{2} dx \qquad (9)$$

Because of the hierarchy between the strong (g) and electromagnetic ($\widetilde{e}$) interactions, we have that $g^2/\widetilde{e}^2 > 1$ and hence $\alpha_{nc} < 0$, indicating that the nucleation of an inhomogeneous condensate G is energetically favored.

To obtain the explicit expression for the condensate solution, we can follow Abrikosov's approach [15] to type II metal superconductivity for the limit situation when the applied field is near the critical value H_{c2}. Close to $\widetilde{H}_C$ the amplitude of the condensate G should be very small and we can neglect the nonlinear term in Eq. (6). Similarly, the G^2 term in (7) can be neglected yielding $\widetilde{B} \approx \widetilde{H}_C$. As a consequence, Eqs. (6)-(7) decouple and G can be found from

$$[\partial_j^2 - \frac{4\pi i}{\widetilde{\Phi}_0} \widetilde{H}_C x \partial_y - 4\pi^2 \frac{\widetilde{H}_C^2}{\widetilde{\Phi}_0^2} x^2 + \frac{1}{\xi^2}] G = 0, \quad j = x, y \qquad (10)$$

where we used the gauge $\widetilde{A}_2 = \widetilde{H}_C x_1$ and defined $\widetilde{\Phi}_0 \equiv 2\pi/\widetilde{e}$, and $\xi^2 \equiv 1/(2\widetilde{e}\widetilde{H}_C - m_M^2) = 1/m_M^2$.

A solution of Eq. (10) can be proposed in the form

$$G(x, y) = e^{-ik_y y} f(x) \qquad (11)$$

with $f(x)$ satisfying the differential equation

$$-f''(x) + 4\pi^2 \frac{\widetilde{H}_C^2}{\widetilde{\Phi}_0^2} (x - x_k)^2 f = \frac{1}{\xi^2} f, \qquad (12)$$

and vanishing at $x \to \pm\infty$. The parameter x_k is $x_k \equiv k_y \widetilde{\Phi}_0 / 2\pi \widetilde{H}_C$. Eqs. (10) and (12) are formally identical to those found for the Abrikosov's vortex state near the critical field H_{c2}. The parameter ξ in (10) is playing the role of the coherence length, i.e. the characteristic length for the variation of G. Increasing the magnetic field strength makes ξ to decrease. The solution of (10), (12) is

$$G_k = \exp[-iky] \exp[-\frac{(x - x_k)^2}{2\xi^2}] \qquad (13)$$

where $k \equiv k_y$. From the experience with conventional type II superconductivity [16] it is known that the inhomogeneous condensate solutions prefer periodic lattice domains to minimize the energy. Then, putting on periodicity in the y-direction with period $\Delta y = b$ restricts the values of k to a discrete set $k = 2\pi n/b$, $n = 1, 2, ...$ This condition implies that we have an infinite set of discrete solutions, and because Eqs. (10) and (12) are linear differential equations, the general solution should be given by their superposition $G(x, y) = \sum C_n G_n$. The superposition of all these Gaussian solutions centered at different x_n constitutes the vortex state needed to solve the instability in the whole space. On the other hand, the discrete values of k imply periodicity in x, since the Gaussian solutions G_n are located at $x_n = \frac{k_n \widetilde{\Phi}_0}{2\pi \widetilde{H}_C} = \frac{n \widetilde{\Phi}_0}{b \widetilde{H}_C}$. Hence, assuming that all G_n enter with equal weight, the periodicity length in the x-direction is $\Delta x = \frac{\widetilde{\Phi}_0}{b \widetilde{H}_C}$. Therefore, the magnetic flux through each periodicity cell in the vortex lattice is quantized $\widetilde{H}_C \Delta x \Delta y = \widetilde{\Phi}_0$, with $\widetilde{\Phi}_0$ being the flux quantum per unit vortex cell. In this semi-qualitative analysis we considered Abrikosov's ansatz of a rectangular lattice (i.e. all the coefficients C_n being equal), but the lattice configuration should be carefully determined from a minimal energy analysis. For the rectangular lattice, we see that the area of the unit cell is $A = \Delta x \Delta y = \widetilde{\Phi}_0 / \widetilde{H}_C$, so decreasing with $\widetilde{H}$.

Substituting with Eq.(13) back into (7) to find a correction to the linear solution for the induced field $\widetilde{B}$, one easily sees that the induced field, while being homogeneous in the z-direction, becomes inhomogeneous in the (x, y)-plane since it depends on the condensate G, which has periodicity on that plane. Therefore $\widetilde{B}$ forms a fluxoid along the z-direction creating a nontrivial topology on the (x, y) plane. Notice that the magnetic flux found in the present paper is qualitatively different from the one found in Ref. [17], where the flux was related to the massive gauge field $\widetilde{G}_\mu^8$. In [17] the flux was due to the existence of nontrivial loops in the order parameter (diquark condensate) space that originated in the transition between the unpaired phase and the CFL phase.

In conclusion, at low $\widetilde{H}$ field, the CFL phase is an insulator, and the $\widetilde{H}$ field just penetrates through it. At sufficiently high $\widetilde{H}$, the condensation of $G^{\pm}$ is triggered inducing the formation of a lattice of magnetic flux tubes and breaking the translational and remaining rotational symmetries. Contrary to the situation in conventional type-II superconductors, where the applied field only penetrates through the flux tubes and with a smaller strength, the vortex state found in this paper has the peculiarity that outside the flux tube the applied field $\widetilde{H}$ totally penetrates the sample, while inside the tubes the magnetic field becomes larger than $\widetilde{H}$. This antiscreening behavior is similar to that of the electroweak system at high magnetic field [7]. Notice that as the $\widetilde{Q}$ photons remain massless in the presence of the condensate G, the $\widetilde{U}(1)_{em}$ symmetry remains unbroken.

A rough estimate of the critical field that produces the magnetic instability at the scale of baryon densities typical of neutron-star cores ($\mu \simeq 200 - 400 MeV$, $\alpha_s(\mu) \simeq 1/3$) gives $\widetilde{H}_C \simeq 9.5 \times 10^{16} G - 3.8 \times 10^{17} G$. Although these are significantly high magnetic fields, they cannot be ruled out as acceptable values for the neutron star core.

We would like to call attention to a possible connection between our results and the problem of the instabilities [18, 19] in gapless CS [20]. As known, a color superconductor can develop chromomagnetic instabilities even in the absence of an external magnetic field. These instabilities may appear after imposing electrical and color neutralities and β equilibrium conditions, and at densities where the s quark mass M_s becomes a relevant parameter. As found first in $g2SC$ [18], and then also in $gCFL$ [19], some charged gluons typically become tachyonic at the onset of the gapless phase. From the outcome of this paper it results clear that although the imaginary Meissner mass of these charged gluons in the gapless phase is not triggered by any external magnetic field, the mechanism we investigated can work here to remove the instability of the gapless phase. In this case we could use the same effective action (3) as a toy model to describe the instabilities of the charged gluons in gapless CS. The main difference will be to consider a negative square Meissner mass from the beginning and to assume that although no external magnetic field is present ($\widetilde{H} = 0$), the induced magnetic field $\widetilde{B}$ can be different from zero (see Eq.(8)) since it is nothing but the rotated electromagnetic field generated by the gluon condensate. Then the instability will be removed through the spontaneous breaking of the spatial rotational symmetry $SO(3) \to SO(2)$ due to vortex nucleation and the induction of a rotated magnetic field perpendicular to the vortex planes. Notice that the new equation for the magnetic field, $2\widetilde{e}G^2 - \widetilde{B} = 0$, can have a nontrivial solution thanks to the antiscreening effect produced by the anomalous magnetic moment contribution $2\widetilde{e}G^2$. This means that the induction of a $\widetilde{B}$ field, together with the topological modification of the medium that goes with it, can be directly linked to the asymptotic freedom of the theory. It remains to be seen what would be the relative field orientation between neighboring domains that minimizes the system free-energy, and whether the generation of G and $\widetilde{B}$ alone is enough to remove the instability of the neutral gluon modes in the gapless phase. If our arguments are corroborated, a color superconducting core could provide a new mechanism to generate and amplify the magnetic fields of compact stars. The idea of using gluon condensates to solve the chromomagnetic instabilities in CS has been also explored in Refs. [21].

It is worth to mention that if the ground state is self-consistently found, by minimizing the free-energy with respect to all the possible condensates: gluon and diquark condensates and the induced rotated magnetic field, the inhomogeneous gluon condensate would produce a back-reaction into the diquark condensate making it inhomogeneous too and quite likely with the same periodicity of the gluon vortex solution.

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